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Finite-time H_∞ filtering of Markov jump systems with incomplete transition probabilities: a probability approach

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Abstract: This paper concerns the finite-time H_∞ filtering of discrete Markov jump system with incomplete transition probabilities which cover the cases of known, uncertain and unknown. To include all possible cases, with the probability viewpoint, a truncated Gaussian distribution is employed to describe them. To ensure the filtering error systems to be finite-time stochastic stable with a prescribed noise attenuation level, sufficient conditions for the H_∞ filter design are yielded in terms of solvability of a set of linear matrix inequalities. A numerical example is given to illustrate the effectiveness of the proposed method.

1 Introduction

Over the past decades, an increasing research activity has been devoted to Markov jump systems (MJSs). Many results about stability, stabilisation, particle filtering, synchronisation control and sliding mode control have been scattered in the literature, see [1–19]. Paralleling to these fruitful theoretical results, many efforts have also been made on the widespread use of this kind of systems, such as aircraft control systems, robotic manipulator systems, power systems and transportation systems [1, 2].

Different from linear systems, the dynamic process of MJSs is governed by a Markov chain, namely, the mode transited from one to another is determined by its transition probabilities which has direct influence on system stability and performance. Under the assumption that the transition probabilities are completely known, a lot of achievements have been reported on stability analysis, controller design and optimisation, see [3–19]. Nevertheless, in engineering, it is difficult or costly to catch the full information of all transition probabilities, which leads that the results established by the idea assumption is restrictive. Taking vertical take-off landing helicopter system in the aerospace industry as an example, the airspeed variation involved in the system matrices are modelled as a Markov chain. Due to the limitation of the measurement equipment, not all the probabilities of the jumps among multiple airspeeds are readily to be measured. Consequently, the obtained transition probabilities may be inaccurate. To conquer this difficulty, some tentative methods have been developed in [20–27]. Concretely speaking, [20, 21] utilise the robust methodology to deal with transition probabilities with norm bounded or polytopic uncertainties. Considering a more realistic situation that the practical transition probability matrix may be partly known, the stability, stabilisation and filtering problems are studied by the authors of [22–26]. With the stochastic viewpoint, an alternative representation of partly known transition probabilities is given in [27–29].

On the other hand, an important problem in the process of control system design is its stability. For MJSs, stochastic stability, moment stability, exponential stability and almost sure stability are well investigated by the authors of [3, 4] over an infinite-time interval. However, the stochastic stability in infinite-time interval may cause large values of the states which are not acceptable in the presence of saturations. To deal with this transient performance of control dynamics, finite-time stability or short-time stability for linear system is proposed in [30–35]. By extending them to stochastic scenario, the finite-time stochastic stability is adopted to study the behaviour of MJSs during a fixed finite-time interval [36–40]. Specially, the finite-time H_∞ filtering of time-delay stochastic jump systems with unbiased estimation is investigated in [36]. On the basis of dynamic observer-based state feedback method, the finite-time H_∞ fuzzy controller design for non-linear jump system with time delays is presented in [37]. Finite-time H_∞ estimation of discrete-time MJSs with time-varying transition probabilities subject to average dwell time switching is discussed in [40]. With the consideration of partly known transition probabilities, the finite-time stochastic stability and stabilisation of discrete MJSs and the finite-time H_∞ filtering non-linear stochastic systems are studied in [38, 39], respectively.

This paper further considers the finite-time filtering of discrete MJSs with incomplete transition probabilities which are assumed to be known, uncertain and unknown. To present these transition probabilities in a unified framework, a stochastic description is proposed in terms of the truncated Gaussian method. On the basis of this description, the main focus is concentrated to the finite-time filter design. To make the filtering error system be stochastically stable with a prescribed H_∞ performance index, on the basis of the stochastic Lyapunov theory and the transition probability property, sufficient conditions for the desired filter design are developed in terms of solvability of a set of linear matrix inequalities. A numerical example is provided to demonstrate the effectiveness of the proposed approach.

Notation: Throughout this paper, $\boldsymbol{M}^{\mathrm{T}}$ represents the transpose of matrix $\boldsymbol{M}$. $\mathbb{Z}^{+}$ denotes the set of positive integers and $N \in \mathbb{Z}^{+}$. The notation $\boldsymbol{X} \leq \boldsymbol{Y} (\boldsymbol{X} < \boldsymbol{Y})$ where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are symmetric matrices, means that $\boldsymbol{X} - \boldsymbol{Y}$ is negative semi-definite (negative definite), respectively. $\boldsymbol{I}$ and 0 represent identity matrix and zero matrix, respectively. $\mathcal{L}_2$ denotes the space of square integrable vector functions of a given dimension over $[0, \infty)$, with norm $E\{||x||_2^2\} = E\{\sum_{k=0}^{\infty} x(k)^{\mathrm{T}} x(k)\,\mathrm{d}t\} < \infty$. ★ denotes the entries of matrices implied by symmetry. $\eta_{\min}$ and $\eta_{\max}$ denote the smallest and the largest eigenvalue of matrix $\boldsymbol{P}$, respectively. ϖ_{ij}^{l} and ϖ_{ij}^{u}

denote the lower and the upper bound of π_{ij}. Notations sup and inf denote the supremum and infimum. Matrices, if not explicitly stated, are assumed to have appropriate dimensions. Finally, the symbol $\text{He}(X)$ is used to represent $(X+X^{\mathrm{T}})$.

2 Preliminaries and problem statement

Consider the following discrete MJSs

$$\begin{cases} x(k+1) &= \boldsymbol{A}(r(k))x(k) + \boldsymbol{B}_1(r(k))w(k) \\ y(k) &= \boldsymbol{C}_1(r(k))x(k) + \boldsymbol{D}_1(r(k))w(k) \\ z(k) &= \boldsymbol{C}_2(r(k))x(k) + \boldsymbol{D}_2(r(k))w(k) \end{cases} \tag{1}$$

where $k \in [0\ N]$, $x(k) \in R^n$ is the state variables, $y(k) \in R^p$ is the measured output, $z(k) \in R^q$ is the signal to be estimated and $w(k) \in R^l$ is the disturbance input which belongs to $\mathcal{L}_2[0, \infty)$. $r(k)$ is a Markov process taking values on the finite set $\mathcal{I} = \{1, 2, \ldots, s\}$ with transition probabilities

$$\pi_{ij} = \Pr(r(k) = j | r(k-1) = i) \tag{2}$$

where π_{ij} denotes the transition probability from mode i at time $k-1$ to mode j at time k. $\boldsymbol{A}(r(k))$, $\boldsymbol{B}_1(r(k))$, $\boldsymbol{C}_1(r(k))$, $\boldsymbol{C}_2(r(k))$, $\boldsymbol{D}_1(r(k))$ and $\boldsymbol{D}_2(r(k))$ are system matrices with appropriate dimensions. when $r(k) = i$, they are abbreviated as $\boldsymbol{A}_i$, $\boldsymbol{B}_{1i}$, $\boldsymbol{C}_{1i}$, $\boldsymbol{C}_{2i}$, $\boldsymbol{D}_{1i}$ and $\boldsymbol{D}_{2i}$.

Practically, in engineering systems, transition probabilities are generally obtained by experiments. Due to the complexity of environment, the measurement cost and the accuracy of equipment, they may not be exactly known. To make the considered problem be readily solvable, robust methodologies are adopted in [21, 22, 25]. Compared with these deterministic strategies, based on a stochastic description, a feasible idea is to utilise their distribution [27, 29]. Omitting the approximation details, the Gaussian probability method is employed to parameterise them. In this case, (2) is modified as follows:

$$\pi_{ij}^{\phi_k} = \Pr(r(k) = j | r(k-1) = i, k) \tag{3}$$

where $\pi_{ij}^{\phi_k}$ $\left(\varpi_{ij}^{\mathrm{l}} \leq \pi_{ij}^{\phi_k} \leq \varpi_{ij}^{\mathrm{u}}\right)$ denotes the transition probability from mode i to mode j and ϕ_k is a set of random variables indexed by Gaussian stochastic process to address transition probabilities varying continuously. The truncated Gaussian probability density function (PDF) $p\left(\pi_{ij}^{\phi_k}\right)$ of $\pi_{ij}^{\phi_k}$ is given below

$$p\left(\pi_{ij}^{\phi_k}\right) = \frac{\left(1/\sqrt{\sigma_{ij}}\right) f\left[\left(\pi_{ij}^{\phi_k} - \mu_{ij}\right)/\sqrt{\sigma_{ij}}\right]}{F\left[\left(\varpi_{ij}^{\mathrm{u}} - \mu_{ij}\right)/\sqrt{\sigma_{ij}}\right] - F\left[\left(\varpi_{ij}^{\mathrm{l}} - \mu_{ij}\right)/\sqrt{\sigma_{ij}}\right]} \tag{4}$$

where $f(\bullet)$ is the PDFof the standard normal distribution, $F(\bullet)$ is the cumulative distribution function of $f(\bullet)$, ϱ_{ij} and σ_{ij} are the known mean and variance of the Gaussian PDF, respectively. Therefore, the transition probability density matrix is defined as

$$\begin{bmatrix} n\left(\varrho_{11}, \sigma_{11}\right) & n\left(\varrho_{12}, \sigma_{12}\right) & \cdots & n\left(\varrho_{1s}, \sigma_{1s}\right) \\ n\left(\varrho_{21}, \sigma_{21}\right) & n\left(\varrho_{22}, \sigma_{22}\right) & \cdots & n\left(\varrho_{2s}, \sigma_{2s}\right) \\ \cdots & \cdots & \cdots & \cdots \\ n\left(\varrho_{s1}, \sigma_{s1}\right) & n\left(\varrho_{s2}, \sigma_{s2}\right) & \cdots & n\left(\varrho_{ss}, \sigma_{ss}\right) \end{bmatrix} \tag{5}$$

where $n\left(\varrho_{ij}, \sigma_{ij}\right) = p\left(\pi_{ij}^{\phi_k}\right)$.

According to the probability theory, the expectation of $\pi_{ij}^{\phi_k}$ is calculated as

$$\begin{aligned} E\left(\pi_{ij}^{\phi_k}\right) &= \int_{\varpi_{ij}^{\mathrm{u}}}^{\varpi_{ij}^{\mathrm{u}}} \pi_{ij}^{\phi_k} p\left(\pi_{ij}^{\phi_k}\right) \mathrm{d}\pi_{ij}^{\phi_k} \\ &= \varrho_{ij} + \frac{f\left(\left[\varpi_{ij}^{\mathrm{l}} - \mu_{ij}\right]/\sqrt{\sigma_{ij}}\right) - f\left(\left[\varpi_{ij}^{\mathrm{u}} - \mu_{ij}\right]/\sqrt{\sigma_{ij}}\right)}{F\left(\left[\varpi_{ij}^{\mathrm{l}} - \mu_{ij}\right]/\sqrt{\sigma_{ij}}\right) - F\left(\left[\varpi_{ij}^{\mathrm{u}} - \mu_{ij}\right]/\sqrt{\sigma_{ij}}\right)} \sqrt{\sigma_{ij}} \end{aligned} \tag{6}$$

Consequently, the desired transition probability matrix is obtained

$$\begin{bmatrix} E\left(\pi_{11}^{\phi_k}\right) & E\left(\pi_{12}^{\phi_k}\right) & \cdots & E\left(\pi_{1s}^{\phi_k}\right) \\ E\left(\pi_{21}^{\phi_k}\right) & E\left(\pi_{22}^{\phi_k}\right) & \cdots & E\left(\pi_{2s}^{\phi_k}\right) \\ \cdots & \cdots & \cdots & \cdots \\ E\left(\pi_{s1}^{\phi_k}\right) & E\left(\pi_{s2}^{\phi_k}\right) & \cdots & E\left(\pi_{ss}^{\phi_k}\right) \end{bmatrix} \tag{7}$$

with $\sum_{j=1}^{s} E\left\{\pi_{ij}^{\phi_k}\right\} = 1$.

Remark 1: According to the above discussion, if $\sigma_{ij} = 0$ ($\sigma_{ij} \neq \infty$), $E\left(\pi_{ij}^{\phi_k}\right)$ is known (uncertain with known bounds). Otherwise ($\sigma_{ij} = \infty$), it is unknown. Therefore, this form is consistent with the form proposed in [23–25].
To facilitate further discussion, the following presentation is utilised:

$$\begin{aligned} \mathcal{I}_k^i &\triangleq \left\{j{:}\sigma_{ij} = 0 \text{ or } \sigma_{ij} \neq \infty\right\}, \\ \mathcal{I}_{uk}^i &\triangleq \left\{j{:}\sigma_{ij} = \infty\right\}. \end{aligned} \tag{8}$$

The aim of this paper is to design a full-order mode-dependent finite-time filter which is given below

$$\begin{cases} x_{\mathrm{f}}(k+1) = \boldsymbol{A}_{\mathrm{f}i} x_{\mathrm{f}}(k) + \boldsymbol{B}_{\mathrm{f}i} y(k) \\ z_{\mathrm{f}}(k) = \boldsymbol{C}_{\mathrm{f}i} x_{\mathrm{f}}(k) + \boldsymbol{D}_{\mathrm{f}i} y(k) \end{cases} \tag{9}$$

where $x_{\mathrm{f}}(k)$ is the filter state, $z_{\mathrm{f}}(k)$ is the filter output. $\boldsymbol{A}_{\mathrm{f}i}$, $\boldsymbol{B}_{\mathrm{f}i}$, $\boldsymbol{C}_{\mathrm{f}i}$ and $\boldsymbol{D}_{\mathrm{f}i}$ are filter gains to be designed.

Remark 2: The difference between a finite-time filtering and a filtering is that the former focuses its attention on the transient behaviour of a system response and the latter is its asymptotic behaviour. In this sense, saturation-induced non-linear effects or safety-critical operative conditions will be avoided in the finite-time case.
Define

$$\hat{x}(k) = \begin{bmatrix} x(k) \\ x_{\mathrm{f}}(k) \end{bmatrix}$$

and $e(k) = z(k) - z_{\mathrm{f}}(k)$, combining (1) with (9), the filtering error dynamic can be written as

$$\begin{cases} \hat{x}(k+1) = \hat{\boldsymbol{A}}_i \hat{x}(k) + \hat{\boldsymbol{B}}_i w(k) \\ e(k) = \hat{\boldsymbol{C}}_i \hat{x}(k) + \hat{\boldsymbol{D}}_i w(k) \end{cases} \tag{10}$$

where

$$\hat{\boldsymbol{A}}_i = \begin{bmatrix} \boldsymbol{A}_i & 0 \\ \boldsymbol{B}_{\mathrm{f}i}\boldsymbol{C}_{1i} & \boldsymbol{A}_{\mathrm{f}i} \end{bmatrix}, \quad \hat{\boldsymbol{B}}_i = \begin{bmatrix} \boldsymbol{B}_i \\ \boldsymbol{B}_{\mathrm{f}i}\boldsymbol{D}_{1i} \end{bmatrix} \tag{11}$$

$$\hat{\boldsymbol{C}}_i = \begin{bmatrix} \boldsymbol{C}_{2i} - \boldsymbol{D}_{\mathrm{f}i}\boldsymbol{C}_{1i} & -\boldsymbol{C}_{\mathrm{f}i} \end{bmatrix}, \quad \hat{\boldsymbol{D}}_i = \boldsymbol{D}_{2i} - \boldsymbol{D}_{\mathrm{f}i}\boldsymbol{D}_{1i} \tag{12}$$

Before discussing the finite-time H_∞ filtering problem, some assumptions and definitions are given below.

Assumption 1: The random variables ϕ_k and $r(k)$ are independent from each other.

Assumption 2: The external disturbance $w(k)$ satisfies the constraint

$$E\left\{\sum_{k=0}^{N} w(k)^{\mathrm{T}} w(k)\right\} \le d \quad (d \ge 0) \tag{13}$$

Definition 1: The discrete MJSs (1) with $w(k) = 0$ is said to be stochastic finite-time stable with respect to (c_1, c_2, $\boldsymbol{R}_i$, N) where $0 < c_1 < c_2$, $\boldsymbol{R}_i > 0$ and $N \in \mathbb{Z}^+$, if

$$E\left\{x^{\mathrm{T}}(0)\boldsymbol{R}_i x(0)\right\} \le c_1 \to E\left\{x^{\mathrm{T}}(k)\boldsymbol{R}_i x(k)\right\} \le c_2 \quad (k = 1, 2, \ldots, N) \tag{14}$$

Definition 2: The discrete MJS (1) is said to be stochastic finite-time boundedness with respect to (c_1, c_2, $\boldsymbol{R}_i$, N, d) where $0 < c_1 < c_2$, $\boldsymbol{R}_i > 0$ and d satisfies (13) if (14) holds.

On the basis of these definitions, the finite-time H_∞ filtering is formulated as follows.

Given a prescribed level of noise attenuation γ ($\gamma > 0$), determine a full-order filter in the form of (9), such that the filtering error system (10) is stochastic finite-time stable and satisfies the following

$$E\left\{\sum_{k=0}^{N} e^{\mathrm{T}}(k)e(k)\right\} \le \gamma^2 E\left\{\sum_{k=0}^{N} w^{\mathrm{T}}(k)w(k)\right\} \tag{15}$$

under zero-initial conditions for any non-zero $w(k)$.

3 Main results

This section presents the main results for the finite-time H_∞ filtering design for discrete MJSs with incomplete transition probabilities. Linear matrix inequality (LMI) conditions are derived to ensure the filtering error system to be finite-time stochastic boundedness.

Lemma 1: The filtering error system (10) is stochastic finite-time boundedness if, for a given scalar μ ($\mu \ge 1$), there exists two sets of symmetric positive-definite matrices $\boldsymbol{P}_i$ and $\boldsymbol{Q}_i$ ($i \in \mathcal{I}$) such that the following inequalities hold:

$$\begin{bmatrix} -\mu\boldsymbol{P}_i & 0 & * \\ 0 & -\boldsymbol{Q}_i & * \\ \mathcal{M}_i\hat{\boldsymbol{A}}_i & \mathcal{M}_i\hat{\boldsymbol{B}}_i & -\mathcal{M}_i \end{bmatrix} < 0 \tag{16a}$$

$$\sup_{i\in\mathcal{I}}\left\{\eta_{\max}\left(\hat{\boldsymbol{P}}_i\right)\right\}c_1 + \sup_{i\in\mathcal{I}} \left\{\eta_{\max}\left(\boldsymbol{Q}_i\right)\right\}d \le \inf_{i\in\mathcal{I}}\left\{\eta_{\min}\left(\hat{\boldsymbol{P}}_i\right)\right\}\mu^{-N}c_2 \tag{16b}$$

where

$$\mathcal{M}_i = \mathcal{P}_k^i + \lambda_k^i\boldsymbol{P}_l, \mathcal{P}_k^i = \sum_{j\in\mathcal{I}_k^i} E\left\{\pi_{ij}^{\phi_k}\right\}\boldsymbol{P}_j,$$
$$\lambda_k^i = 1 - \sum_{j\in\mathcal{I}_k^i} E\left\{\pi_{ij}^{\phi_k}\right\},$$
$$\hat{\boldsymbol{P}}_i = \boldsymbol{R}_i^{-(1/2)}\boldsymbol{P}_i\boldsymbol{R}_i^{-(1/2)}, \quad l \in \mathcal{I}_{uk}^i.$$

Proof: Choose the following quadratic Lyapunov functional candidate for the error system (10):

$$V(k) := V(\hat{x}(k), r(k) = i) = \hat{x}^{\mathrm{T}}(k)\boldsymbol{P}_i\hat{x}(k) \tag{17}$$

where $\boldsymbol{P}_i > 0$ for each i. Then, $V(k+1)$ is calculated as

$$\begin{aligned} E\{V(k+1)\} &= \left\{\hat{x}^{\mathrm{T}}(k)\hat{\boldsymbol{A}}_i^{\mathrm{T}} + w^{\mathrm{T}}(k)\hat{\boldsymbol{B}}_i^{\mathrm{T}}\right\} \\ &\quad\times\left(\sum_{j=1}^{s} E\left\{\pi_{ij}^{\phi_k}\right\}\right)\boldsymbol{P}_j\left\{\hat{\boldsymbol{A}}_i\hat{x}(k) + \hat{\boldsymbol{B}}_i w(k)\right\} \\ &= \begin{bmatrix} \hat{x}(k) \\ w(k) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \hat{\boldsymbol{A}}_i^{\mathrm{T}}\sum_{j=1}^{s} E\left\{\pi_{ij}^{\phi_k}\right\}\boldsymbol{P}_j\hat{\boldsymbol{A}}_i & \hat{\boldsymbol{A}}_i^{\mathrm{T}}\sum_{j=1}^{s} E\left\{\pi_{ij}^{\phi_k}\right\}\boldsymbol{P}_j\hat{\boldsymbol{B}}_i \\ \star & 0 \end{bmatrix} \\ &\quad \begin{bmatrix} \hat{x}(k) \\ w(k) \end{bmatrix} \end{aligned} \tag{18}$$

Due to $\boldsymbol{P}_i$ coupling with $E\left\{\pi_{ij}^{\phi_k}\right\}$, (18) is formulated in bilinear form, which is difficult to be solved. To separate this interconnection, the transition probability property is made full use of. Namely, employing the fact $\sum_{j\in\mathcal{I}_k^i} E\left\{\pi_{ij}^{\phi_k}\right\} + \sum_{j\in\mathcal{I}_{uk}^i} E\left\{\pi_{ij}^{\phi_k}\right\} = 1$, one has $\left[\left(\sum_{j\in\mathcal{I}_{uk}^i} E\left\{\pi_{ij}^{\phi_k}\right\}\right)/\left(1 - \sum_{j\in\mathcal{I}_k^i} E\left\{\pi_{ij}^{\phi_k}\right\}\right)\right] = 1$. For convenience, some abbreviated expressions are given as $\beta_k^i = \left[\left(\sum_{j\in\mathcal{I}_{uk}^i} E\left\{\pi_{ij}^{\phi_k}\right\}\right)/\left(1 - \sum_{j\in\mathcal{I}_k^i} E\left\{\pi_{ij}^{\phi_k}\right\}\right)\right]$, $\lambda_k^i = 1 - \sum_{j\in\mathcal{I}_k^i} E\left\{\pi_{ij}^{\phi_k}\right\}$ and $\mathcal{P}_k^i = \sum_{j\in\mathcal{I}_k^i} E\left\{\pi_{ij}^{\phi_k}\right\}\boldsymbol{P}_j$. Taking a transformation to (18), one has

$$\begin{aligned} E\{V(k+1)\} &= \begin{bmatrix} \hat{x}(k) \\ w(k) \end{bmatrix}^{\mathrm{T}} \\ &\quad\times\left\{\beta_k^i\begin{bmatrix} \hat{\boldsymbol{A}}_i^{\mathrm{T}}\left(\lambda_k^i\boldsymbol{P}_l + \mathcal{P}_k^i\right)\hat{\boldsymbol{A}}_i & \hat{\boldsymbol{A}}_i^{\mathrm{T}}\left(\lambda_k^i\boldsymbol{P}_l + \mathcal{P}_k^i\right)\hat{\boldsymbol{B}}_i \\ \star & \hat{\boldsymbol{B}}_i^{\mathrm{T}}\left(\lambda_k^i\boldsymbol{P}_l + \mathcal{P}_k^i\right)\hat{\boldsymbol{B}}_i \end{bmatrix}\right\} \\ &\quad\times\begin{bmatrix} \hat{x}(k) \\ w(k) \end{bmatrix} \end{aligned} \tag{19}$$

On the other hand, referring to Schur complement to (16a), it yields

$$\begin{bmatrix} \hat{\boldsymbol{A}}_i^{\mathrm{T}}\left(\mathcal{P}_k^i + \lambda_k^i\boldsymbol{P}_l\right)\hat{\boldsymbol{A}}_i & \hat{\boldsymbol{A}}_i^{\mathrm{T}}\left(\mathcal{P}_k^i + \lambda_k^i\boldsymbol{P}_l\right)\hat{\boldsymbol{B}}_i \\ 0 & \hat{\boldsymbol{B}}_i^{\mathrm{T}}\left(\mathcal{P}_k^i + \lambda_k^i\boldsymbol{P}_l\right)\hat{\boldsymbol{B}}_i \end{bmatrix} < \begin{bmatrix} \mu\boldsymbol{P}_i & 0 \\ \star & \boldsymbol{Q}_i \end{bmatrix} \tag{20}$$

Taking (20) to (19), it follows

$$\begin{aligned} E\{V(k+1)\} &< \left\{\mu\hat{x}^{\mathrm{T}}(k)\boldsymbol{P}_i\hat{x}(k) + w^{\mathrm{T}}(k)\boldsymbol{Q}_i w(k)\right\} \\ &\le E\left\{\mu V(k) + \sup_{j\in\mathcal{I}}\left\{\eta_{\max}(\boldsymbol{Q}_i)\right\}w^{\mathrm{T}}(k)w(k)\right\} \\ &= \mu E\{V(k)\} + \sup_{j\in\mathcal{I}}\left\{\eta_{\max}(\boldsymbol{Q}_i)\right\}E\left\{w^{\mathrm{T}}(k)w(k)\right\} \end{aligned} \tag{21}$$

Thus

$$\begin{aligned}
&E\{V(k)\}\\
&\quad<\mu E\{V(k-1)\}+\sup_{j\in\mathcal{I}}\{\eta_{\max}(\boldsymbol{Q}_i)\}E\{w^{\mathrm{T}}(k-1)w(k-1)\}\\
&\quad<\mu^2E\{V(k-1)\}+\sup_{j\in\mathcal{I}}\{\eta_{\max}(\boldsymbol{Q}_i)\}E\{w^{\mathrm{T}}(k-1)w(k-1)\\
&\quad+\mu w^{\mathrm{T}}(k-2)w(k-2)\}\\
&\quad\cdots\\
&\quad<\mu^kE\{V(0)\}+\sup_{j\in\mathcal{I}}\{\eta_{\max}(\boldsymbol{Q}_i)\}E\left\{\sum_{h=0}^{k-1}\mu^{k-h-1}w^{\mathrm{T}}(h)w(h)\right\}\\
&\quad<\mu^NE\{V(0)\}+\sup_{j\in\mathcal{I}}\{\eta_{\max}(\boldsymbol{Q}_i)\}\mu^Nd
\end{aligned}\tag{22}$$

Let $\hat{\boldsymbol{P}}_i=\boldsymbol{R}_i^{-(1/2)}\boldsymbol{P}_iR_i^{-(1/2)}$ and note $E\{\hat{x}^{\mathrm{T}}(0)\boldsymbol{R}_i\hat{x}(0)\}\le c_1$, one has

$$\begin{aligned}
E\{V(0)\}&=E\{\hat{x}^{\mathrm{T}}(0)\boldsymbol{P}_i\hat{x}(0)\}\\
&=E\left\{\hat{x}^{\mathrm{T}}(0)\boldsymbol{R}_i^{(1/2)}\hat{\boldsymbol{P}}_i\boldsymbol{R}_i^{(1/2)}\hat{x}(0)\right\}\\
&\le\sup_{i\in\mathcal{I}}\{\eta_{\max}(\hat{\boldsymbol{P}}_i)\}E\{\hat{x}^{\mathrm{T}}(0)\boldsymbol{R}_i\hat{x}(0)\}\le\sup_{i\in\mathcal{I}}\{\eta_{\max}(\hat{\boldsymbol{P}}_i)\}c_1
\end{aligned}\tag{23}$$

On the other hand

$$\begin{aligned}
E\{V(k)\}&=E\left\{\hat{x}^{\mathrm{T}}(k)\boldsymbol{R}_i^{(1/2)}\hat{\boldsymbol{P}}_i\boldsymbol{R}_i^{(1/2)}\hat{x}(k)\right\}\\
&\ge\inf_{i\in\mathcal{I}}\{\eta_{\min}(\hat{\boldsymbol{P}}_i)\}E\{\hat{x}^{\mathrm{T}}(k)\boldsymbol{R}_i\hat{x}(k)\}
\end{aligned}\tag{24}$$

Combining with (22)–(24), the following formula is derived:

$$\begin{aligned}
&E\{\hat{x}^{\mathrm{T}}(k)\boldsymbol{R}_i\hat{x}(k)\}\\
&\quad<\frac{\sup_{i\in\mathcal{I}}\{\eta_{\max}(\hat{\boldsymbol{P}}_i)\}\mu^Nc_1+\sup_{i\in\mathcal{I}}\{\eta_{\max}(\boldsymbol{Q}_i)\mu^Nd\}}{\inf_{i\in\mathcal{I}}\{\eta_{\min}(\hat{\boldsymbol{P}}_i)\}}
\end{aligned}\tag{25}$$

From (16b), $E\{\hat{x}^{\mathrm{T}}(k)\boldsymbol{R}_i\hat{x}(k)\}<c_2$ $(k=1,\ 2,\ \ldots,\ N)$ can be guaranteed. □

Remark 3: In the proceeding of derivation, to separate the coupling among unknown transition probabilities and Lyapunov variables, the transition probability property is made full use of.

On the basis of Lemma 1, a sufficient condition for the filtering error system (10) to be stochastic finite-time boundedness with the prescribed H_∞ performance index is given in the following lemma.

Lemma 2: The filtering error system (10) is stochastic finite-time boundedness with the prescribed H_∞ performance index γ, if, for a given constant scalar μ $(\mu\ \ge 1)$, there exist a set of symmetric positive-definite matrices $\boldsymbol{P}_i$ $(i\in\mathcal{I})$ such that the following inequalities hold:

$$\begin{bmatrix}-\mu\boldsymbol{P}_i & 0 & * & *\\ 0 & -\mu^{-N}\gamma^2\boldsymbol{I} & * & *\\ \mathcal{M}_i\hat{\boldsymbol{A}}_i & \mathcal{M}_i\hat{\boldsymbol{B}}_i & -\mathcal{M}_i & \\ \hat{\boldsymbol{C}}_i & \hat{\boldsymbol{D}}_i & 0 & -\boldsymbol{I}\end{bmatrix}<0 \tag{26a}$$

$$\sup_{i\in\mathcal{I}}\{\eta_{\max}(\hat{\boldsymbol{P}}_i)\}c_1+\mu^{-N}\gamma^2d\le\inf_{i\in\mathcal{I}}\{\eta_{\min}(\hat{\boldsymbol{P}}_i)\}\mu^{-N}c_2 \tag{26b}$$

where $\mathcal{M}_i$ and $\hat{\boldsymbol{P}}_i$ is the same as that of Lemma 1.

Proof: On the basis of (26a), the following inequality holds:

$$\begin{bmatrix}-\mu\boldsymbol{P}_i & 0 & *\\ 0 & -\mu^{-N}\gamma^2\boldsymbol{I} & *\\ \mathcal{M}_i\hat{\boldsymbol{A}}_i & \mathcal{M}_i\hat{\boldsymbol{B}}_i & -\mathcal{M}_i\end{bmatrix}<0 \tag{27}$$

According to Lemma 1, the filtering error system (10) to be stochastic finite-time boundedness can be guaranteed by replacing $\boldsymbol{Q}_i$ with $\mu^{-N}\gamma^2\boldsymbol{I}$. Thus, we only need to prove that (15) holds under zero-value initial condition. Choose the same Lyapunov function as Lemma 1, then it follows

$$\begin{aligned}
&E(V(k+1))-\mu E(V(k))+E\left(e^{\mathrm{T}}(k)e(k)-\mu^{-N}\gamma^2w^{\mathrm{T}}(k)w(k)\right)\\
&\quad=\begin{bmatrix}\hat{x}(k)\\ w(k)\end{bmatrix}^{\mathrm{T}}\Pi_i\begin{bmatrix}\hat{x}(k)\\ w(k)\end{bmatrix}
\end{aligned}\tag{28}$$

where (see equation at bottom of the page)

Taking the similar lines as that of Lemma 1 to deal with the completely unknown transition probabilities, (28) can be further rewritten as (see (29))

where $\Xi_i=\lambda_k^i\boldsymbol{P}_l+\mathcal{P}_k^i$.
Combining (29) with (26a), one has

$$\begin{aligned}
&E(V(k+1))-\mu E(V(k))\\
&\quad+E\left(e^{\mathrm{T}}(k)e(k)-\mu^{-N}\gamma^2w^{\mathrm{T}}(k)w(k)\right)<0
\end{aligned}\tag{30}$$

which is also rewritten as

$$\begin{aligned}
&E(V(k+1))<\mu E(V(k))\\
&\quad-E\left(e^{\mathrm{T}}(k)e(k)\right)+\mu^{-N}\gamma^2E\left(w^{\mathrm{T}}(k)w(k)\right)
\end{aligned}\tag{31}$$

Similarly to (22), we can obtain

$$\begin{aligned}
E(V(k))<&\mu^kE(V(0))-\sum_{h=0}^{k-1}\mu^{k-h-1}E\left(e^{\mathrm{T}}(h)e(h)\right)\\
&+\mu^{-N}\gamma^2\sum_{h=0}^{k-1}\mu^{k-h-1}E\left(w^{\mathrm{T}}(h)w(h)\right)
\end{aligned}\tag{32}$$

$$\Pi_i=\begin{bmatrix}\hat{\boldsymbol{A}}_i^{\mathrm{T}}\sum_{j=1}^{s}E\left\{\pi_{ij}^{\phi_k}\right\}\boldsymbol{P}_j\hat{\boldsymbol{A}}_i+\hat{\boldsymbol{C}}_i^{\mathrm{T}}\hat{\boldsymbol{C}}_i-\mu\boldsymbol{P}_i & \hat{\boldsymbol{A}}_i^{\mathrm{T}}\sum_{j=1}^{s}E\left\{\pi_{ij}^{\phi_k}\right\}\boldsymbol{P}_j\hat{\boldsymbol{B}}_i+\hat{\boldsymbol{C}}_i^{\mathrm{T}}\hat{\boldsymbol{D}}_i\\ \star & \hat{\boldsymbol{B}}_i^{\mathrm{T}}\sum_{j=1}^{s}E\left\{\pi_{ij}^{\phi_k}\right\}\boldsymbol{P}_j\hat{\boldsymbol{B}}_i+\hat{\boldsymbol{D}}_i^{\mathrm{T}}\hat{\boldsymbol{D}}_i-\mu^{-N}\gamma^2\boldsymbol{I}\end{bmatrix}$$

$$\begin{aligned}
&E(V(k+1))-\mu E(V(k))+E\left(e^{\mathrm{T}}(k)e(k)-\mu^{-N}\gamma^2w^{\mathrm{T}}(k)w(k)\right)\\
&\quad=\begin{bmatrix}\hat{x}(k)\\ w(k)\end{bmatrix}^{\mathrm{T}}\left\{\beta_k^i\begin{bmatrix}\hat{\boldsymbol{A}}_i^{\mathrm{T}}\Xi_i\hat{\boldsymbol{A}}_i+\hat{\boldsymbol{C}}_i^{\mathrm{T}}\hat{\boldsymbol{C}}_i-\mu\boldsymbol{P}_i & \hat{\boldsymbol{A}}_i^{\mathrm{T}}\Xi_i\hat{\boldsymbol{B}}_i+\hat{\boldsymbol{C}}_i^{\mathrm{T}}\hat{\boldsymbol{D}}_i\\ \star & \hat{\boldsymbol{B}}_i^{\mathrm{T}}\Xi_i\hat{\boldsymbol{B}}_i-\mu^{-N}\gamma^2\boldsymbol{I}\end{bmatrix}\right\}\begin{bmatrix}\hat{x}(k)\\ w(k)\end{bmatrix}
\end{aligned}\tag{29}$$

Under the zero-value initial condition, one has

$$\sum_{h=0}^{k-1}\mu^{k-h-1}E\big(e^{\mathrm{T}}(h)e(h)\big) < \mu^{-N}\gamma^2\sum_{h=0}^{k-1}\mu^{k-h-1}E\big(w^{\mathrm{T}}(h)w(h)\big) \quad (33)$$

Due to the fact that $\mu \geq 1$, we can obtain

$$\begin{aligned}\sum_{h=0}^{N}E\big(e^{\mathrm{T}}(h)e(h)\big) &\leq \sum_{h=0}^{N}\mu^{N-h}E\big(e^{\mathrm{T}}(h)e(h)\big)\\ &< \mu^{-N}\gamma^2\sum_{h=0}^{N}\mu^{N-h}E\big(w^{\mathrm{T}}(h)w(h)\big) \leq \gamma^2\sum_{h=0}^{N}E\big(w^{\mathrm{T}}(h)w(h)\big)\end{aligned} \quad (34)$$

□

Since it is difficult to solve (26b) directly, a tractable condition formulated in terms of LMIs is given in the following lemma.

Lemma 3: The filtering error system (10) is stochastic finite-time stable with the prescribed H_∞ performance index γ if, for a constant scalar μ ($\mu \geq 1$), there exist positive scalars ψ_1, ψ_2, a set of matrices $\boldsymbol{G}_i$ and $\boldsymbol{P}_i > 0$ ($i \in \mathcal{I}$) such that the following inequalities hold:

$$\begin{bmatrix} -\mu\boldsymbol{P}_i & 0 & * & * \\ 0 & -\mu^{-N}\gamma^2\boldsymbol{I} & * & * \\ \boldsymbol{G}_i\hat{\boldsymbol{A}}_i & \boldsymbol{G}_i\hat{\boldsymbol{B}}_i & \mathrm{He}(-\boldsymbol{G}_i)+\mathcal{M}_i & * \\ \hat{\boldsymbol{C}}_i & \hat{\boldsymbol{D}}_i & 0 & -\boldsymbol{I}\end{bmatrix} < 0 \quad (35a)$$

$$\begin{bmatrix} \mu^{-N}(-\psi_1 c_2 + \gamma^2 d) & \sqrt{c_1}\psi_2 \\ \star & -\psi_2 \end{bmatrix} < 0 \quad (35b)$$

$$\psi_1\boldsymbol{R}_i \leq \boldsymbol{P}_i \leq \psi_2\boldsymbol{R}_i \quad (35c)$$

where $\mathcal{M}_i$ is the same as that of *Lemma* 1.

Proof: The derivation of (35a) from (26a) is based on the method proposed in [41]. For the second part, the derivation (35b) and (35c) from (26b) is given below. According to $\hat{\boldsymbol{P}}_i = \boldsymbol{R}_i^{-(1/2)}\boldsymbol{P}_i\boldsymbol{R}_i^{-(1/2)}$, let $\psi_1\boldsymbol{R}_i \leq \boldsymbol{P}_i \leq \psi_2\boldsymbol{R}_i$, then we have

$$\psi_2 c_1 + \mu^{-N}\gamma^2 d < \mu^{-N}\psi_1 c_2 \quad (36)$$

which is just (35b). □

Remark 4: Via introducing slack variables $\boldsymbol{G}_i$, contrast to Lemma 2, there is no interconnection among $\boldsymbol{P}_i$, $\hat{\boldsymbol{A}}_i$ and $\hat{\boldsymbol{B}}_i$. Moreover, by restricted $\boldsymbol{P}_i$ as a special structure, (26b) is expressed in terms of linear matrix inequality. Although this transformation may introduce some conservativeness, it simplifies the filter design. In future, we will consider how to reduce this conservativeness.

On the basis of Lemma 3, sufficient conditions for the finite-time filter design are given in the following theorem.

Theorem 1: Given positive scalars μ ($\mu \geq 1$) and γ. If there exist approximate matrices

$$\boldsymbol{P}_i = \begin{bmatrix} P_{i11} & \star \\ P_{i12} & P_{i22}\end{bmatrix} > 0, \quad \boldsymbol{G}_i = \begin{bmatrix} G_{i11} & G_{i2} \\ G_{i21} & G_{i2}\end{bmatrix},$$

ψ_1, ψ_2, a_{fi}, b_{fi}, c_{fi} and d_{fi} ($i \in \mathcal{I}$) such that the following matrix inequalities

$$\begin{bmatrix} -\mu\Theta_{i11} & 0 & * & * \\ 0 & -\mu^{-N}\gamma^2\boldsymbol{I} & * & * \\ \Theta_{i31} & \Theta_{i32} & \Theta_{i33} & * \\ \Theta_{i41} & \Theta_{i42} & 0 & -\boldsymbol{I}\end{bmatrix} < 0 \quad (37a)$$

$$\begin{bmatrix} \mu^{-N}(-\psi_1 c_2 + \gamma^2 d) & \sqrt{c_1}\psi_2 \\ \star & -\psi_2 \end{bmatrix} < 0 \quad (37b)$$

$$\psi_1\boldsymbol{R}_i \leq \boldsymbol{P}_i \leq \psi_2\boldsymbol{R}_i \quad (37c)$$

where

$$\Theta_{i11} = \begin{bmatrix} P_{i11} & \star \\ P_{i12} & P_{i22}\end{bmatrix}, \quad \Theta_{i31} = \begin{bmatrix} G_{i11}\boldsymbol{A}_i + b_{fi}\boldsymbol{C}_{1i} & a_{fi} \\ G_{i21}\boldsymbol{A}_i + b_{fi}\boldsymbol{C}_{1i} & a_{fi}\end{bmatrix}$$

$$\Theta_{i32} = \begin{bmatrix} G_{i11}\boldsymbol{B}_i + b_{fi}\boldsymbol{D}_{1i} \\ G_{i21}\boldsymbol{B}_i + b_{fi}\boldsymbol{D}_{1i}\end{bmatrix},$$

$$\in \Theta_{i33} = \mathrm{He}(-\boldsymbol{G}_i) + \sum_{j\in\mathcal{I}_k^i} E\left\{\pi_{ij}^{\phi_k}\right\}\boldsymbol{P}_j^{\phi_k} + \lambda_k^i\boldsymbol{P}_l$$

$$\Theta_{i41} = \begin{bmatrix} \boldsymbol{C}_{2i} - d_{fi}\boldsymbol{C}_{1i} & -c_{fi}\end{bmatrix}, \quad \Theta_{i42} = \boldsymbol{D}_{2i} - d_{fi}\boldsymbol{D}_{1i}, \quad l \in \mathcal{I}_{uk}^i.$$

Then, the filter error system (10) is stochastic finite-time boundedness and has a prescribed H_∞ index γ. Moreover, the desired filter parameters are given by $\boldsymbol{A}_{fi} = G_{i2}^{-1}a_{fi}$, $\boldsymbol{B}_{fi} = G_{i2}^{-1}b_{fi}$, $\boldsymbol{C}_{fi} = c_{fi}$ and $\boldsymbol{D}_{fi} = d_{fi}$.

Proof: Taking the structures of

$$\boldsymbol{P}_i = \begin{bmatrix} P_{i11} & \star \\ P_{i12} & P_{i22}\end{bmatrix} \text{ and } \boldsymbol{G}_i = \begin{bmatrix} G_{i11} & G_{i2} \\ G_{i21} & G_{i2}\end{bmatrix}$$

into Lemma 3 and choosing $a_{fi} = G_{i2}\boldsymbol{A}_{fi}$, $b_{fi} = G_{i2}\boldsymbol{B}_{fi}$, $c_{fi} = \boldsymbol{C}_{fi}$ and $d_{fi} = \boldsymbol{D}_{fi}$, then the proof is completed. □

Remark 5: Note that the conditions given in this theorem for given $\boldsymbol{R}_i$, c_1, N, μ are linear to other variables. Therefore, the optimal γ can be researched by replacing $\delta = \gamma^2$ in (37a)–(37c).

4 Numerical example

Consider the following class of discrete-time MJSs (1) with four operation modes and the following data

$$\boldsymbol{A}_1 = \begin{bmatrix} 0.5 & -0.55 \\ 0.6 & -0.2\end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} 0.5 & -0.5 \\ -0.4 & -0.2\end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0.6 & 0.45 \\ 0.5 & 0.2\end{bmatrix}, \quad \boldsymbol{A}_4 = \begin{bmatrix} 0.45 & 0.4 \\ -0.6 & 0.5\end{bmatrix}$$

$$\boldsymbol{B}_1 = \begin{bmatrix} -0.8 & 0.5\end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{B}_2 = \begin{bmatrix} 0.8 & 0.3\end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{B}_3 = \begin{bmatrix} 0.3 & 0.2\end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{B}_4 = \begin{bmatrix} 0 & 1.2\end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{C}_{11} = \begin{bmatrix} 1.5 & -0.5\end{bmatrix}, \quad \boldsymbol{C}_{12} = \begin{bmatrix} -0.9 & 0.9\end{bmatrix},$$

$$\boldsymbol{C}_{13} = \begin{bmatrix} 1.1 & 0.1\end{bmatrix}, \quad \boldsymbol{C}_{14} = \begin{bmatrix} 1.5 & 0.5\end{bmatrix}$$

$$\boldsymbol{C}_{21} = \begin{bmatrix} 1.2 & -0.9\end{bmatrix}, \quad \boldsymbol{C}_{22} = \begin{bmatrix} 0.05 & 0.5\end{bmatrix},$$

$$\boldsymbol{C}_{23} = \begin{bmatrix} 0.01 & 0.5\end{bmatrix}, \quad \boldsymbol{C}_{24} = \begin{bmatrix} 0.5 & 0.5\end{bmatrix}$$

$$\boldsymbol{D}_{11} = 0.8, \quad \boldsymbol{D}_{12} = 1.2, \quad \boldsymbol{D}_{13} = 0.8,$$

$$\boldsymbol{D}_{14} = 0.8, \quad \boldsymbol{D}_{21} = 0.8, \quad \boldsymbol{D}_{22} = 1.1,$$

$$\boldsymbol{D}_{23} = 0.6, \quad \boldsymbol{D}_{24} = 0.8.$$

with the initial value $x_0 = \begin{bmatrix} 0.3 & 0.2 \end{bmatrix}^{\mathrm{T}}$. Borrowed from [27], the measured transition probability matrix with a probability approach is represented as

$$\begin{bmatrix} n(0.3, 0) & n(0.2, 0) & n(0.1, 0) & n(0.4, 0) \\ n(0.3, \infty) & n(0.2, \infty) & n(0.3, \infty) & n(0.2, \infty) \\ n(0.1, 0.01) & n(0.1, \infty) & n(0.5, \infty) & n(0.3, 0) \\ n(0.2, 0.01) & n(0.2, \infty) & n(0.1, \infty) & n(0.5, \infty) \end{bmatrix} \quad (38)$$

According to the matrix, it can be seen that the covariances of some elements tend to ∞ which means the corresponding element is completely unknown. If the approach proposed in [20] is employed, where the uncertain transition probabilities are treated as unknown, the above transition probability matrix is reduced to

$$\begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ ? & ? & 0.3 & 0.2 \\ ? & ? & ? & 0.3 \\ ? & ? & ? & ? \end{bmatrix} \quad (39)$$

However, by taking mathematical expectation to (38), the approximated transition probability matrix is given below

$$\begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ ? & ? & 0.3 & 0.2 \\ 0.10248 & ? & ? & 0.3 \\ 0.19881 & ? & ? & ? \end{bmatrix} \quad (40)$$

Comparing these two matrices, it can be seen that the latter has more information than the former.

With the obtained transition probability matrix (40), the finite-time filter problem of system (1) over the fixed time interval $\begin{bmatrix} 0 & 10 \end{bmatrix}$ is tested. Setting $\mu = 1.05$, $c_1 = 0.1$, $c_2 = 1$, $d = 0.9$

$$\boldsymbol{R}_i = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

and $N = 10$, by solving (37a)–(37c), the optimal γ is 1.6219 and the filtering parameters are given below

$$\boldsymbol{A}_{\mathrm{f}1} = \begin{bmatrix} 0.0920 & -0.1990 \\ -0.0224 & 0.0378 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{f}2} = \begin{bmatrix} 0.1059 & -0.1145 \\ -0.0680 & -0.0769 \end{bmatrix},$$

$$\boldsymbol{A}_{\mathrm{f}3} = \begin{bmatrix} -0.0112 & 0.1048 \\ 0.0030 & -0.0283 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{f}4} = \begin{bmatrix} -0.0161 & 0.0855 \\ 0.0151 & -0.0803 \end{bmatrix}$$

$$\boldsymbol{B}_{\mathrm{f}1} = \begin{bmatrix} -0.1398 \\ -0.4235 \end{bmatrix}, \quad \boldsymbol{B}_{\mathrm{f}2} = \begin{bmatrix} 0.1668 \\ 0.0974 \end{bmatrix},$$

$$\boldsymbol{B}_{\mathrm{f}3} = \begin{bmatrix} -1.5012 \\ 0.4421 \end{bmatrix}, \quad \boldsymbol{B}_{\mathrm{f}4} = \begin{bmatrix} -0.4617 \\ 0.4335 \end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{f}1} = \begin{bmatrix} -0.1450 \\ 0.4000 \end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{C}_{\mathrm{f}2} = \begin{bmatrix} -0.4764 \\ 0.2046 \end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{C}_{\mathrm{f}3} = \begin{bmatrix} 0.0153 \\ -0.1427 \end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{C}_{\mathrm{f}4} = \begin{bmatrix} 0.1803 \\ -0.4875 \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{D}_{\mathrm{f}1} = 0.8216, \quad \boldsymbol{D}_{\mathrm{f}2} = 0.3844, \quad \boldsymbol{D}_{\mathrm{f}3} = 0.0736, \quad \boldsymbol{D}_{\mathrm{f}4} = 0.5985.$$

Thus, one possible mode of evolution, the filter state response curves $x_{\mathrm{f}1}(k)$ and $x_{\mathrm{f}2}(k)$, the curves $z(k)$ and $z_{\mathrm{f}}(k)$, and filter finite-time evolution curves $x_{\mathrm{f}}^{\mathrm{T}}(k)\boldsymbol{R}_i x_{\mathrm{f}}(k)$ are given in Figs. 1–4, respectively.

From the above response curves, it can be seen that the designed filter ensures the finite-time stochastic stability with the optimised H_∞ performance level.

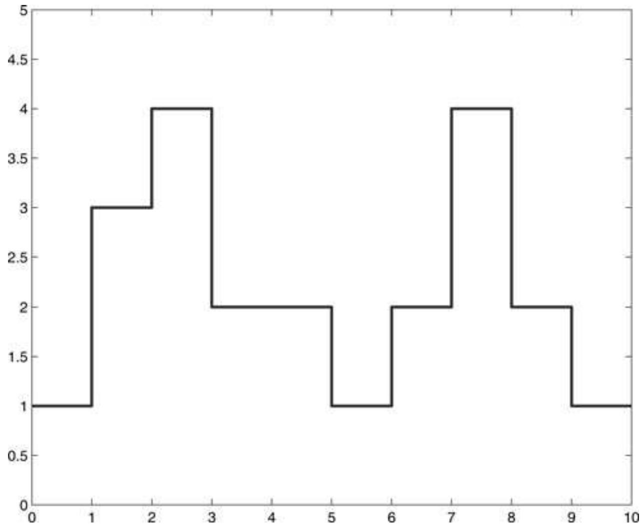

Fig. 1 *One possible mode of evolution*

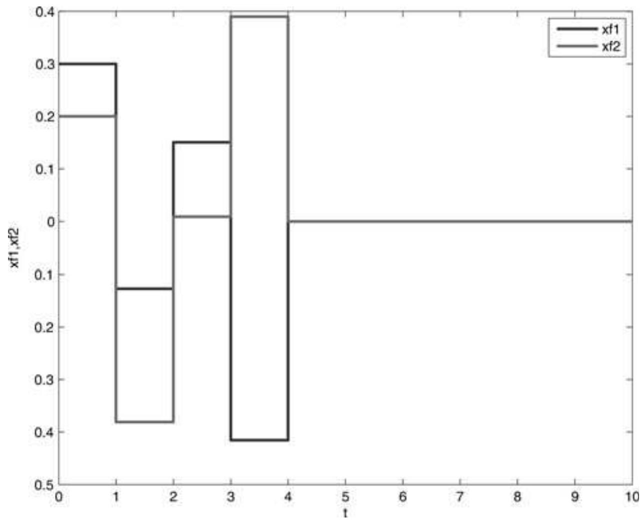

Fig. 2 *Filter state response curves* $x_{f1}(k)$ *and* $x_{f2}(k)$

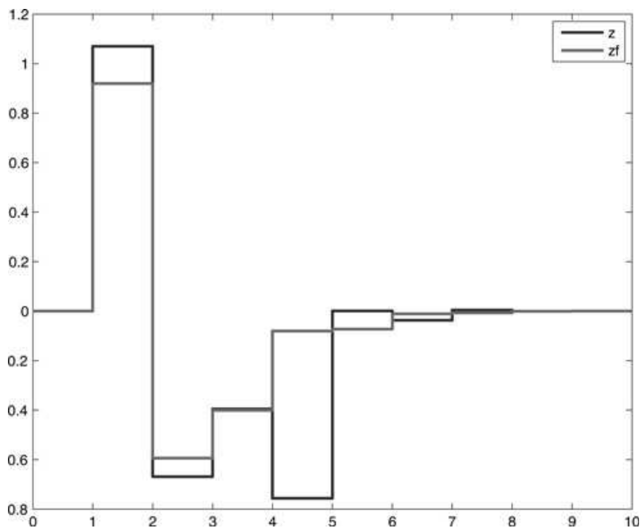

Fig. 3 *Curves* $z(k)$ *and* $z_f(k)$

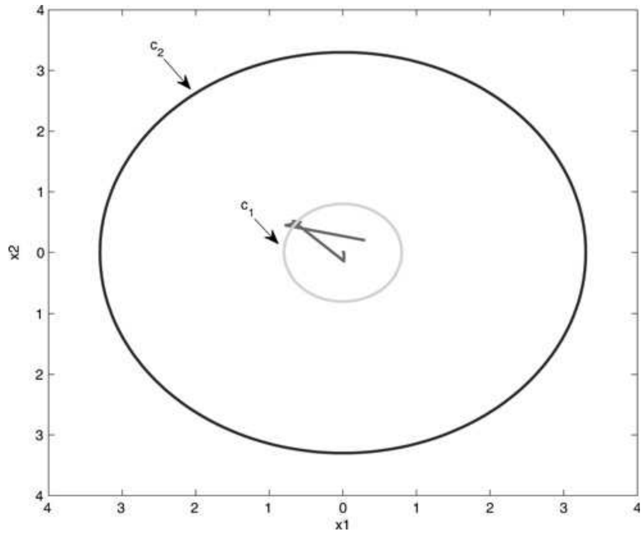

Fig. 4 *Filter finite-time evolution curves* $x_f^T(k)\boldsymbol{R}_i x_f(k)$

5 Conclusions

This paper considers the finite-time H_∞ filtering of discrete MJSs with incomplete transition probabilities represented by the Gaussian probability method. Sufficient conditions are established in the framework of LMIs to guarantee the stochastic finite-time boundedness and a prescribed H_∞ index of the filtering error system. The effectiveness of the developed method is shown by a numerical example.

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